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TWO METHODS OF DETERMINING DAMPING OF FREE, DAMPED SYSTEMS.(U)

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U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

TWO METHODS OF DETERMINING DAMPING OF FREE, DAMPED SYSTEMS

By

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INTRODUCTION

This memorandum discusses a comparison that was made of two methods of determining damping coefficients of a free, damped system. Although both methods have been used in the calibration of the electronic circuitry that was used in the tests, they may also be used in mechanical damping tests of plates, beams, structures and materials.

EXPERIMENTAL PROCEDURE

In each method, three sets of oscillograms were taken for each 1/3 octave band center frequency beginning at 200 cps and ending at 16,000 cps.

Method 1

Reference (a) gives a detailed description of the instrumentation used for this method. Figure 1 shows the setup of this instrumentation. The decade attenuator in the circuit was used to calibrate the vertical scale (dbs./cm.) of the oscilloscope. The horizontal scale of the oscilloscope was then calibrated in seconds per centimeter. These two calibrations had to be done using a continuous wave for each 1/3 octave band center frequency. After the oscilloscope was calibrated, a pulse equivalent to the 1/3 octave band center frequency was transmitted. This pulse and the decay in the system were recorded on the storage oscilloscope. A Poloroid photograph was taken of each oscillogram. The entire procedure was repeated for each 1/3 octave band center frequency.

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The opinions expressed are those of the author(s) and not necessarily the official views of the Laboratory.

Method 2

Figure 2 shows the block diagram of the instrumentation used in this method. It should be noted that the equipment is identical to that used in Method 1, except that the decade attenuator is removed. A pulse was transmitted through the system and the decay of the electronic circuit was recorded on the storage oscilloscope. A Polaroid photograph of the oscillogram was taken. This simplified procedure was used for each 1/3 octave band center frequency.

In each method, three sets of data were taken and analyzed, and the final results were averaged.

DETERMINATION OF DAMPING COEFFICIENTS

Method 1

Reference (b) outlines the procedure for computing the decay rate of a system with a single degree of freedom. In place of the logaten described in reference (b), a logarithmic amplifier and a decade attenuator were used for the purpose of obtaining the vertical scale calibration constant.

In this method, using the circuitry of Figure 1, the decay rate of the system in db/sec was found from the relationship:

$$D = \frac{my}{nx} \tag{1}$$

where: m = the calibration constant of the vertical scale of the oscilloscope, db/cm

n = the calibration constant of the horizontal scale of the oscilloscope, sec/cm

y = the vertical amplitude of the pulse, cm

x = the distance on the abscissa from the pulse to the end of the decay, cm

From reference (a), the percentage of critical camping, $\% \, c/c_c$, was found from the relationship:

$$\% \, c/c_c = 1.84 \, D/f \tag{2}$$

where: D = the decay rate, db/sec

f = the 1/3 octave band center frequency, cps

Method 2

Reference (d) outlines the procedure for the calculation of the damping ratio for use with the instrumentation of Figure 2. The damping ratio can be determined by the relationahip:

$$Dr \simeq \frac{0.75}{n} \tag{3}$$

where n = the number of cycles of motion in the length of record required for the amplitude of the envelope of the motion to decrease to 1% of its initial value.

From reference (f), the differential equation of motion of a viscous - damped, linear, single degree of freedom second order system is:

$$m \frac{d^2x}{dt^2} + c\frac{dx}{dt} + kX = 0 \tag{4}$$

where: m = the mass, $\frac{\text{lb -sec}^2}{\text{in.}}$

x = the dependent variable, inches

t = the independent variable, seconds

c = the viscous damping constant, in $\frac{\text{lb. - sec.}}{\text{inch}}$

k = the spring constant, in $\frac{\text{lbs.}}{\text{inch.}}$

For sub-critical damping, the solution of equation (4) becomes (from reference (f)):

$$X = e^{-\frac{c}{2m}t}\left[c_1 \cos qt + c_2 \sin qt\right] \tag{5}$$

where: x = the instantaneous displacement of the mass in inches.

c = the viscous damping constant, $\frac{\text{lb. - sec.}}{\text{inch}}$

m = the mass, $\frac{\text{lb. - sec}^2}{\text{inch}}$

t = the time, seconds

e = the base of natural logarithms, 2.71828

c_1 = an arbitrary constant

c_2 = an arbitrary constant

and

$$q = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

where: k = the spring constant, $\frac{\text{lbs.}}{\text{inch.}}$

m = the mass, $\frac{\text{lb.-sec}^2}{\text{inch.}}$

c = the viscous damping constant $\frac{\text{lb. - sec.}}{\text{inch.}}$

The rate of decay is controlled by the decay constant, k. It is shown, in reference (e), that the decay constant can be represented by the relationship:

$$k = \frac{c}{2m} \tag{6}$$

where c = the viscous damping constant, $\frac{\text{lb.-sec.}}{\text{inch}}$

m = the mass, $\frac{\text{lb.-sec}^2}{\text{inch}}$

In reference (d), it is shown that:

$$\frac{c}{2m} = D_r \omega_n \tag{7}$$

Therefore: $k = D_r \omega_n$

where: D_r = the damping ratio

ω_n = the circular frequency, radians/second

From reference (f):

$$\omega = 2 \pi f \tag{8}$$

where: f = the frequency, cps

$\pi = 3.1416$

Therefore, it can be seen that:

$$k = 2 \pi f D_r \tag{9}$$

where: $\pi = 3.1416$

f = the 1/3 octave band center frequency, cps

D_r = the damping ratio

From reference (f), the decay rate (db/sec) is given by the relationship:

$$D = 20 \log_{10} e^k = (20 \log_{10} e) \; k = 8.68 \; k \tag{10}$$

where: k = the decay constant, lbs./inch

Therefore: $D = 2 \pi (8.68) f D_r$

-or-

$$D = 54.6 \; f \; D_r \tag{11}$$

where: f = the 1/3 octave band center frequency, cps

D_r = the damping ratio

From reference (a), the per cent of critical damping is given by the relationship:

$$\% \; c/c_c = 1.84 \; D/f$$

where: D = the decay rate, db/sec

f = the 1/3 octave band center frequency, cps

-or-

$$\% \; c/c_c = 2\pi(1.84)\;(8.68)\;D_r \tag{12}$$

Therefore:

$$\% \; c/c_c = 100 \; D_r$$

where: D_r = the damping ratio

DISCUSSION OF THE TWO METHODS

Method 1

This method involved the calibration of the vertical scale of the oscilloscope using the decade attenuator for each center frequency of the filter band. Also, the horizontal scale had to be calibrated for each frequency. This involved transmitting a continuous wave through the system each time. To find the decay rate (db/sec), equation (1) was used. To use this equation, the calibration constants of both the horizontal and vertical scales must be known. The vertical amplitude of the pulse had to be measured, as well as the horizontal distance from the pulse to the end of the decay. The percentage of critical damping was found by equation (2).

Method 2

This method, described in reference (d), required only that the oscilloscope be initially calibrated. No calibration constants were needed. The only measurement needed was the amplitude of the pulse. From this, one could find the location of the point in the decay where the amplitude had decreased to 1 per cent of the amplitude of the pulse.

The only other information needed was the number of cycles that occur between the two points. This can easily be counted from the oscillogram.

The damping ratio was found from equation (3). The decay rate (db/sec) was found from equation (11), and the percentage of critical damping was found from equation (12).

RESULTS

Reference (g) gives the relative error as:

$$E_{rel} = \frac{e}{y} \tag{13}$$

where: e = the error or deviation from the reference in this case

y = the reference value

Reference (g) also shows the percentage error as:

$$\%E = \frac{100\ e}{y} \tag{14}$$

Table I shows the average decay rates for each frequency for each method. The percent error is also shown using Method 1 as a reference. Figure 3 shows a comparison of the average decay rates for the two methods.

Table II shows the average percentage of critical damping for each frequency for each method. Figure 4 shows a plot of per cent of critical damping for the two methods.

DISCUSSION OF RESULTS

Referring to Figure 3, one can see that the two methods give results that are very close. The per cent error is low for frequencies from 500 cps to 16,000 cps. For these same frequencies, the per cent of critical damping for Method 2 closely follow the per cent of critical damping of Method 1.

The amount of time spent in Method 2 was considerably less than that of Method 1. Also, the experimental procedure of Method 2 was much simpler than that of Method 1.

CONCLUSIONS

Method 2 was found to be much more direct and less time-consuming than Method 1. The results were very good for frequencies above and including 500 cps. It is believed that Method 2, can be used with confidence and with little chance of error for those frequencies from 500 cps to 16,000 cps. One must be more careful for frequencies below 500 cps. In the writer's opinion, Method 1 would be better for frequencies below 500 cps. Both methods may be used; to conduct mechanical damping tests of plates, beams, structures, and materials; and to calibrate the instrumentation. It is recommended that Method 1 be used for systems with a very small amount of damping.

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LIST OF REFERENCES

(a) H. N. Phelps, Jr. and M. F. Borg, "Calibration of Instrumentation for Vibration and Damping Tests," USL Technical Memorandum No. 933-236-63, 22 August 1963.

(b) LT(jg) J. E. Barger, USN, "An Experimental Determination of the Degree of Damping of Structures," USL Technical Memorandum No. 1210-94-59, 17 June 1959.

(c) Geiger and Hamme (Consultants in Acoustics), "The Concept of Damping of Structure Borne Sound and Vibration for Noise Control," Contract No. NObs-73549, NS-713-212 (SF 013-11-01, Task 1353), Request No. USN-1, April 1961, AS AD 290487.

(d) Trapp and Forney, "WADC-University of Minnesota Conference on Acoustical Fatigue," WADC Technical Report 59-676, Project No. 7360, March 1961, ASTIA No. 266374.

(e) Myklestad, N. O., "Vibration Analysis," McGraw-Hill Book Company, Inc., New York, 1944.

(f) Den Hartog, J. P., "Mechanical Vibrations," McGraw-Hill Book Company, Inc., New York, 1956.

(g) G. E. F. Sherwood and A. E. Taylor, "Calculus," Prentice-Hall, Inc., Englewood Cliffs, 1959.

TABLE I.			
1/3 Octave Center Band Frequency. (cps).	Average Decay Rate. (db./sec.) Method 1.	Average Decay Rate. (db./sec.) Method 2.	% Error (Method 1 as reference).
16000	38000	36283	-4.52%
12500	29600	28667	-3.51%
10000	21934	22533	2.73%
8000	17834	18700	4.83%
6400	13833	13233	-4.34%
5000	13083	12867	-1.655%
4000	9366	9617	2.68%
3200	7507	6927	-7.27%
2500	5597	5560	-0.663%
2000	4353	4177	-4.03%
1600	3643	3420	-6.12%
1250	2847	2793	-1.90%
1000	2135	2172	1.73%
800	1905	1847	-2.99%
640	1347	1540	14.35%
500	1200	1218	1.50%
400	544	1065	49.00%
320	504	845	67.50%
250	445	807	81.30%
200	542	689	27.10%

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TABLE II.		
1/3 Octave Center Cand Frequency (cps).	%C/Cc. Method 1.	%C/Cc. Method 2.
16000	4.37%	4.17%
12500	4.36%	4.22%
10000	4.04%	4.14%
8000	4.11%	4.29%
6400	3.98%	3.81%
5000	4.81%	4.73%
4000	4.29%	4.42%
3200	4.32%	3.98%
2500	4.12%	4.09%
2000	4.0[illegible]%	3.84%
1600	4.19%	3.93%
1250	4.19%	4.11%
1000	3.93%	4.00%
800	4.38%	4.24%
640	3.87%	4.43%
500	4.42%	4.48%
400	2.51%	4.90%
320	2.89%	4.87%
250	3.27%	5.94%
200	4.99%	6.35%

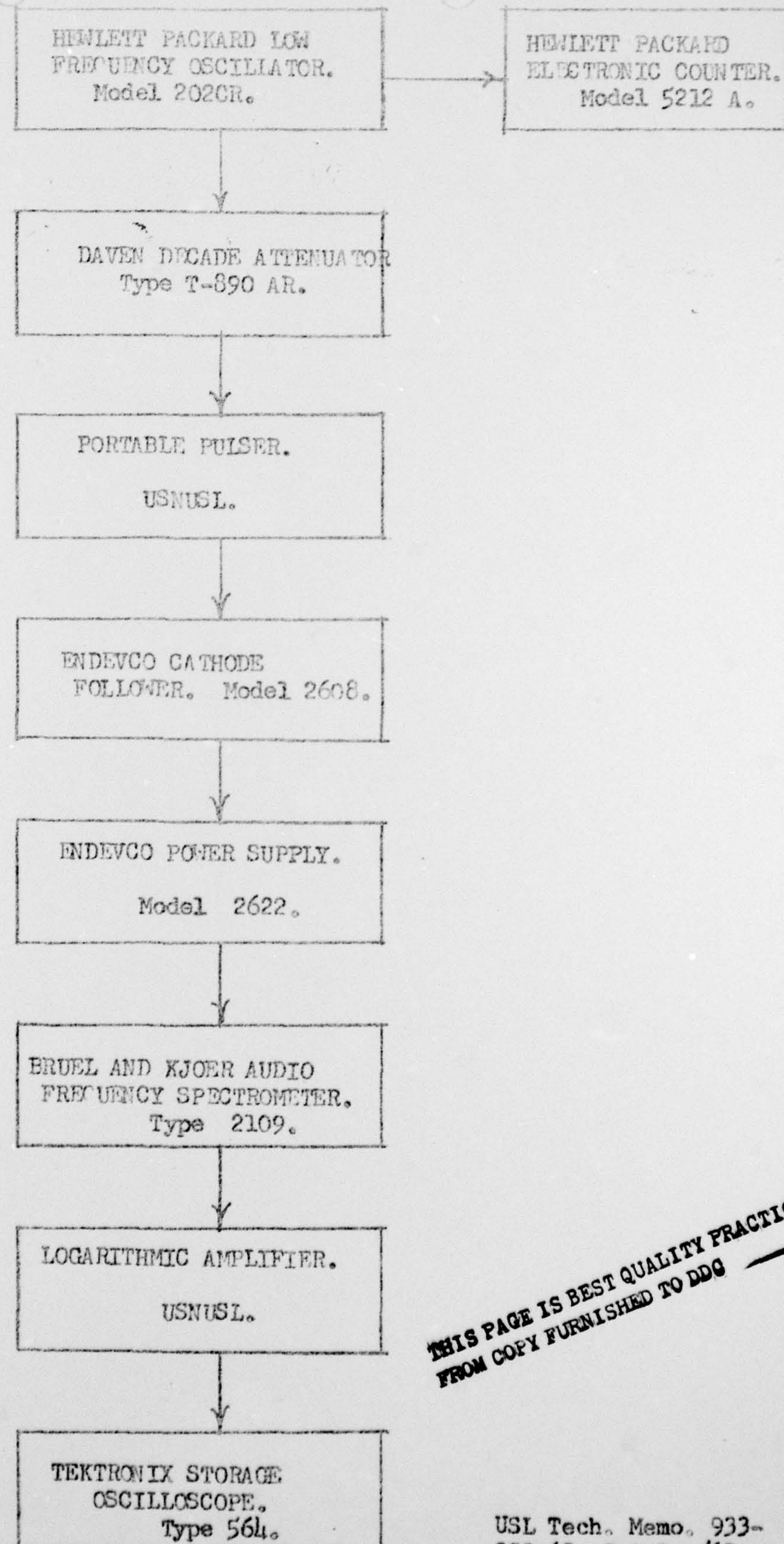


FIGURE 1.
INSTRUMENTATION USED IN METHOD 1.

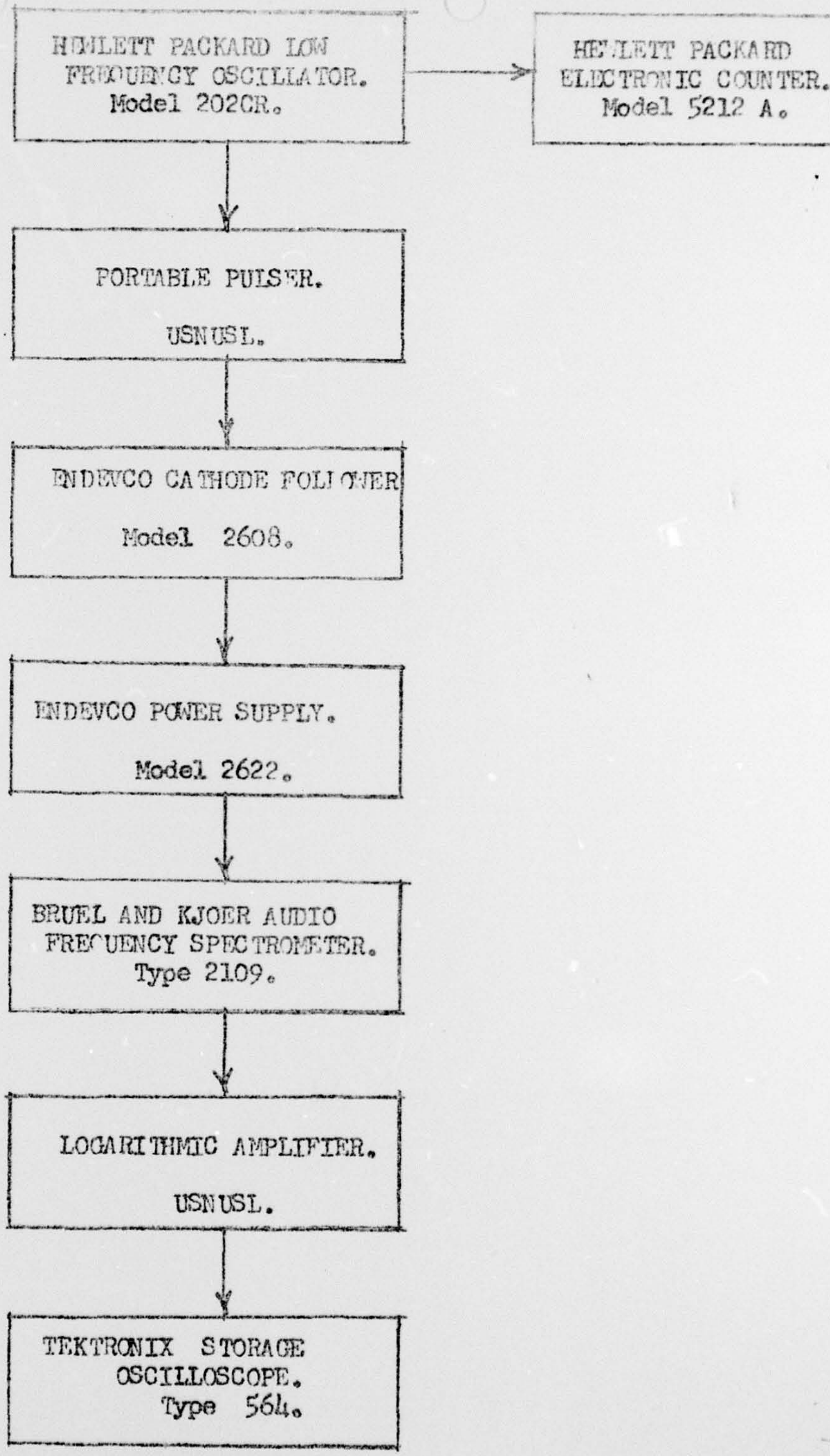


Figure 2.
INSTRUMENTATION USED IN METHOD 2.

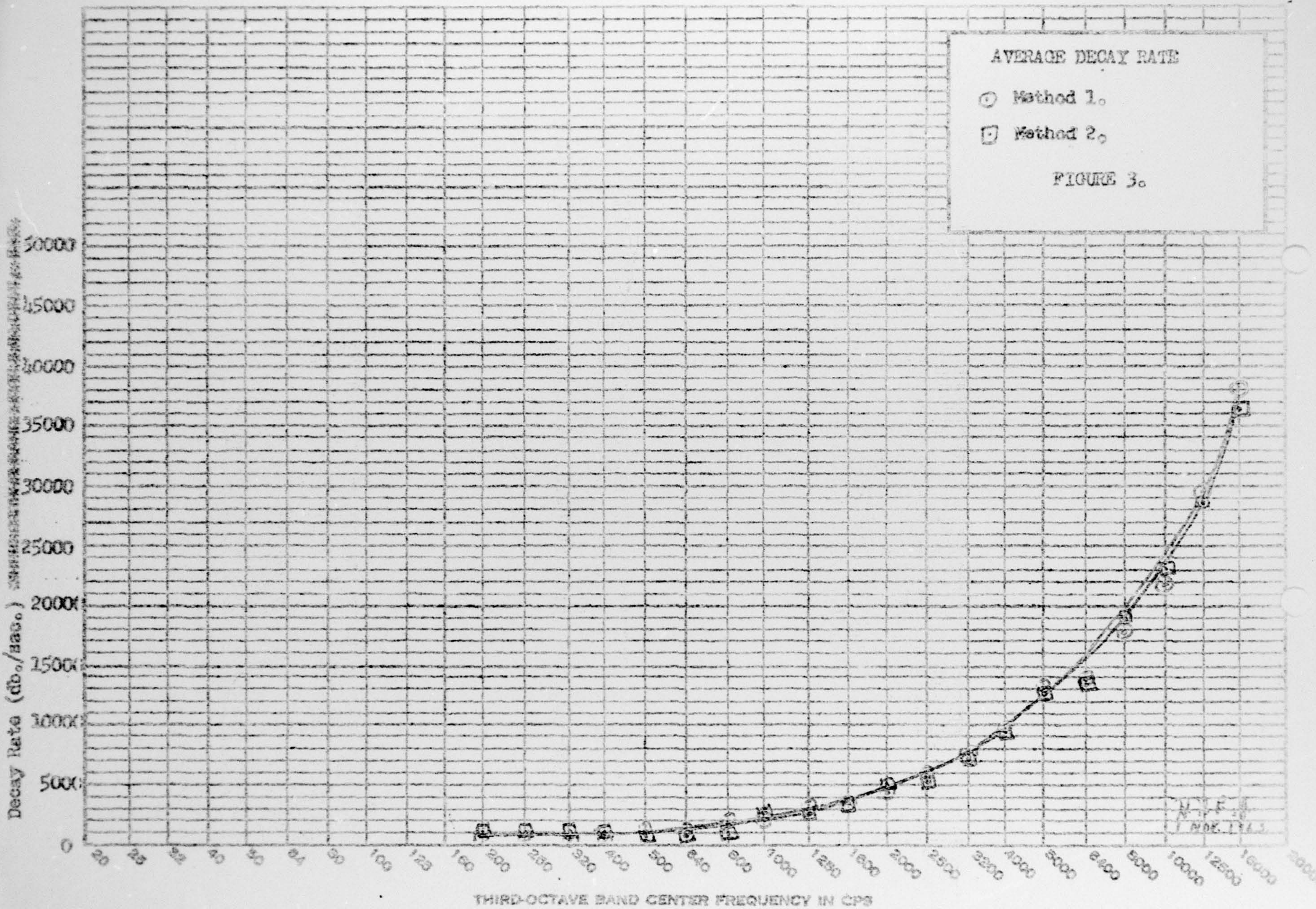
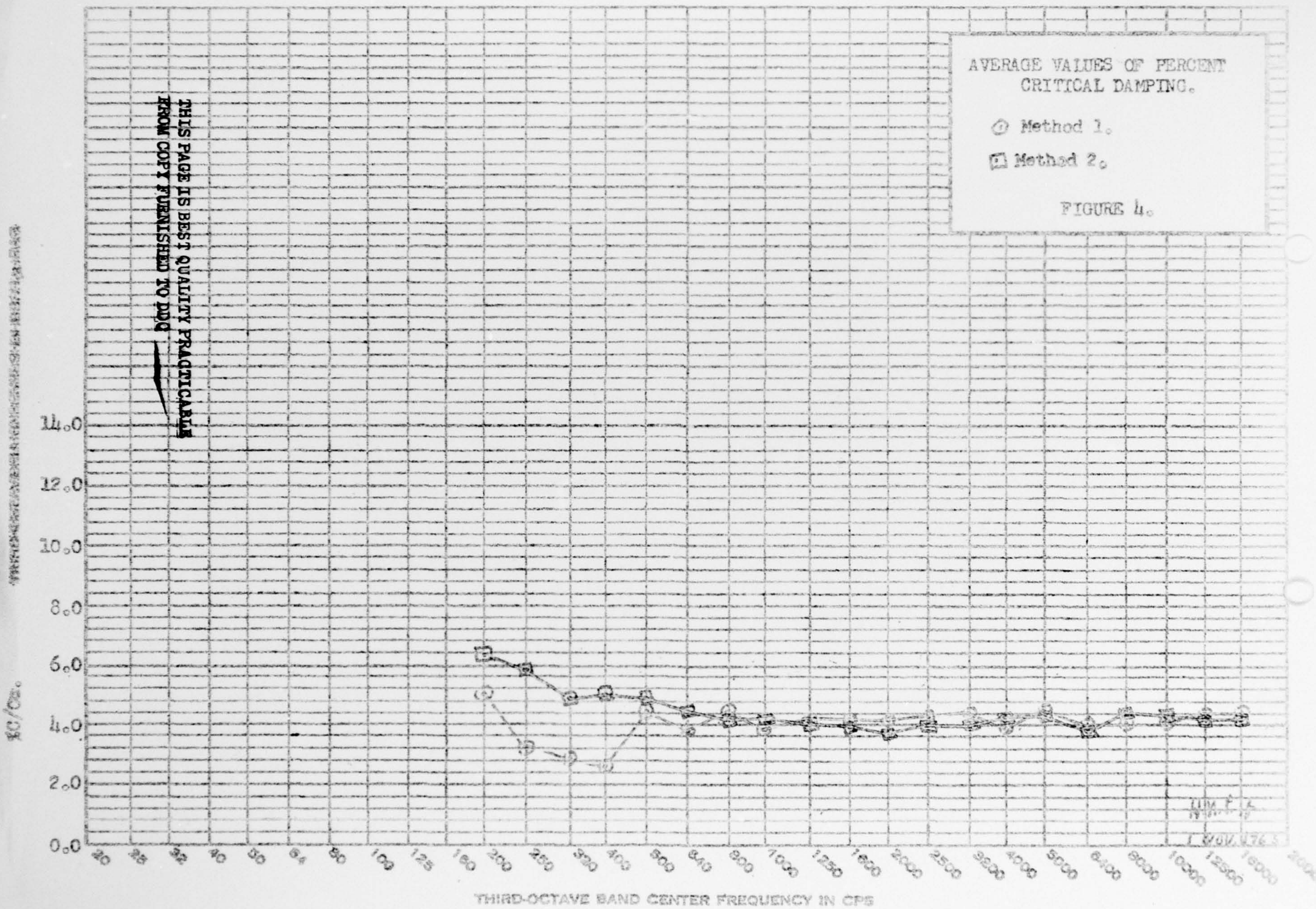

AVERAGE DECAY RATE
Method 1.
Method 2.
Decay Rate (db./sec.)
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
20 25 32 40 50 64 80 100 125 160 200 250 320 400 500 640 800 1000 1250 1600 2000 2500 3200 4000 5000 6400 8000 10000 12500 16000 20000
THIRD-OCTAVE BAND CENTER FREQUENCY IN CPS

FIGURE 3.


AVERAGE VALUES OF PERCENT CRITICAL DAMPING.
Method 1.
Method 2.
FIGURE 4.
%C/Cc.
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
20
25
32
40
50
64
80
100
125
160
200
250
320
400
500
640
800
1000
1250
1600
2000
2500
3200
4000
5000
6400
8000
10000
12500
16000
20000
THIRD-OCTAVE BAND CENTER FREQUENCY IN CPS

Distribution List

Code 100
Code 101
Code 900
Code 900A
Code 900C
Code 907
Code 922
Code 930
Code 930-S (3)
Code 932
Code 933
A. Donn Cobb
G. T. Adkins
H. N. Phelps, Jr.
Code 904
Code 902
Code 904.2 (5)

External

BUSHIPS (Code 689C) (2)
BUSHIPS (Code 688E) (2)
USNASL (Code 9370)